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# A Consensus Algorithm Based on Delay Independent/Dependent Two Methods

**Mehdi R. Vázquez**

*School of Electrical and Computer Engineering, University of Tehran, Iran*

## ABSTRACT

In this paper, we study the distributed finite time event triggered two-part consistent control for multi-agent systems on networks. It is proved that when the adversarial network is balanced by a spanning tree, a sufficient and necessary condition is established to ensure that all agents reach the same size but opposite sign consistent value, and a consensus algorithm based on the delay independent/dependent methods is proposed. This algorithm can be used when the agent part accesses its neighbor's signal. Compared with the asymptotic control algorithm, the proposed control method has better anti-interference performance and convergence speed. The simulation results verify the validity of the theoretical results.

## KEYWORDS

Multi-agent systems; Finite-time consensus; Communication delay; Lyapunov-Razumikhin theorem; Delay-dependent consensus.

## 1. Introduction

In the past few decades, distributed cooperative control of multi-agent systems has been extensively studied[1]. In these systems, the main objective is designing local controllers for each agent to achieve an appropriate group behavior. Among different types of group coordination such as formation, flocking, coverage, rendezvous, etc, consensus over multi-agent systems has a wide variety of applications. Moreover, a consensus scheme is often applied to other types of multi-agent system coordination. Consensus control aims to achieve an agreement among agents' states, by designing controllers using the states or outputs of their neighbors.

There are two significant challenges associated with the performance of multi-agent systems that are mainly inevitable; The presence of time-delay in the communication between agents, and the rate of consensus achievement. Classical stability analysis approaches consider the steady-state performance of the systems and study their stability during an infinite time interval; In which convergence of the system trajectories to the equilibrium is guaranteed. However, in many practical systems such as communication networks, robotic control systems, etc, the transient performance of the system is of great importance. We seek control objectives in a finite-time interval, which is faster and more precise compared to stability in the traditional sense of

Lyapunov. Consider a cooperative robotic system that fulfills several tasks ranging from approaching and grasping a given object to some more complicated and precise tasks like minimally invasive robotic surgeries. In these applications, reaching the goal in a finite-time is indispensable. Important criteria in such task performance are the convergence rate and ultimate bound of the consensus error. The strategy which brings up finite settling, error vanishing time can be beneficial; since it guarantees that the task is performed exactly as commanded. The finite-time stabilization concept originates from optimal control problems that pertain to the dynamical systems whose operation time is limited to a fixed finite-time interval[2]. Some more recent work in this field include[3-6].

The work of [7] analyzed the finite-time convergence of a nonlinear consensus algorithm for multi-agent networks with unknown inherent nonlinear dynamics. For this aim, the authors proposed a stability tool based on a generalized comparison lemma and showed that the proposed nonlinear consensus algorithm can guarantee finite-time convergence if the directed switching interaction graph forms a spanning tree at each time interval. The master-slave finite-time synchronization control problem using an adaptive-fuzzy approach was considered in [8] for the networked teleoperation systems. They developed a new nonsingular fast terminal sliding mode (NFTSM) to provide faster convergence and higher precision compared with the linear hyperplane and classic terminal-sliding mode (TSM).

In [9], distributed adaptive finite-time continuous control algorithms for leaderless consensus of nonlinear mechanical multi-agent systems under an undirected graph are suggested and transient performance in terms of convergence rates is analyzed. Authors of [10] considered finite-time synchronization between two complex dynamical networks by using periodically intermittent control. The work reported in [11] investigates the problem of finite-time consensus tracking for a class of multiple uncertain mechanical systems under switching topologies, uncertainties and input saturations. In [12], the authors studied the targeted agreement problem of a group of Lagrangian systems for fixed and switching graphs. Each system targets a convex set and the objective is to reach a coordinate agreement towards the sets.

In none of the above references, the problem of the presence of time-delays in the communication between the agents has been considered. However, in the real world, many practical systems experience time delays because of the finite speed of information processing/transmission between agents and limited channels bandwidth. It is well known that a time delay may cause undesirable dynamic behaviors such as oscillation, performance degradation, and instability in the system. Consider a networked multi-agent system in which agents communicate with each other through links that suffer from inevitable communication delays. Besides, in teleoperation systems that use communication networks for interaction among master and slave robots, time-delays are unavoidable. Since the whole stability of the system is influenced by time delays, one of the main tasks in such systems is reducing their adverse effects. Thus, it is necessary to study multi-agent systems in the presence of time delays. There are few results in the context of multi-agent systems that consider time delay in the communication links between agents (see e.g., [13-16]).

## 2. Preliminaries

None of the mentioned works studied finite-time control, and in all of them, linear dynamic networks are considered. In[17], the synchronization analysis of the networked manipulators operating on an under-actuated dynamic platform in the presence of communication delays was performed. There are also some few works considering the finite-time stability of time-delayed linear systems (see[18]) but none of them are applied for control of nonlinear multi-agent systems. In our recent work[19], the finite-time consensus in the presence of time-delays was considered. However, the nature of delayed consensus is apparent in this work, and consensus between all agents does not happen synchronously, i.e., the consensus error is defined as $x_i(t) - x_j(t - \tau)$, where $\tau$ is the communication delay and i, j are agent indices. Moreover, the delays are assumed to be known.

Our strategy to compensate the time-delay effect is providing a scheme. in which each agent has a local consensus control algorithm that uses its own signals and delayed signals of its neighbors. An agent does not have to possess all the neighbor signals and only part of them is sufficient. Furthermore, the communication time-delays can be time-varying, non-uniform, non-symmetric and even unknown. Besides, the proposed algorithm works for a large class of nonlinear systems. These characteristics extend the generalization of the proposed consensus control algorithm. It is conspicuous that establishing the multi-agent consensus is much more difficult in the finite-time sense, especially when there is a delay in the transmissions. The reason is that the dynamics of the cooperative system in the presence of delays are more complicated and much more restrictive conditions should be confirmed in the case of finite-time consensus.

The contributions of this paper are threefold: 1) Design of a control law to guarantee the finite-time consensus in the presence of communication time-delays. In this regard, the sufficient conditions to establish the delay-independent/dependent consensus are presented. The difference between these two approaches is that in the delay-dependent one, the consensus criteria depend on the upper-bound of delays and therefore is less conservative. The suggestion of these new control strategies is possible by defining a novel consensus error vector, which does not read a direct difference of agent's signals, unlike the common consensus approaches. 2) Providing conditions by which the algorithms work in the presence of partial access to the neighboring agent signals, which is a prevalent condition in many practical multi-agent systems, where the finite-time consensus is considered. 3) There are no strong restrictions about the communication time-delays between agents. They do not need to be constant, uniform, symmetric or even known. Instead, knowing an upper-bound on all of them is required. To the best of author' knowledge, it is the first time to extend the solution of finite-time consensus problem, which applies to a large class of nonlinear systems in the presence of communication delays.

The paper is organized as follows: In Section 4, some preliminaries including a short description of graph theory and required finite-time algorithm lemmas, the problem statement, system description, and its properties are presented. The delay-independent and delay-dependent consensus strategies are provided in Section 5. Simulation results and conclusive points are given in Sections 6 and 7, respectively.

## 3. Consensus Control Strategy

A communication graph G is denoted by G = (V, E, A), where V = {1, ..., N} is a finite nonempty node set, E V V⊆ × is the edge set of pairs of nodes and A is the adjacency matrix. The edge set E, represents the communication links between the nodes. The ordered pair ( j i, ) ∈E shows that node i obtains information from node j. In other words, j is the neighbor of i. The neighbor set of node i is defined as Ni ={j j i(, ) ∈E}. We assume that G does not contain any self loops. The adjacency matrix G is defined as aij = 1 if (i, j) ∈E and aij = 0 otherwise.

The in-degree of a node is the number of edges that this node is the ending point for them. Similarly, the out-degree is the number of edges that this node is the beginning point for them. If the in-degree and out-degree are equal for all the nodes, the graph is said to be balanced. Assume N

In this paper, we consider both undirected and directed communication graph topologies. We assume the undirected graph is connected and the directed one is balanced and strongly-connected. Besides, since there are packet losses and asynchronous clocks in the communications, there unavoidably exist delays in the transmission of agent signals. Therefore, the graph in this paper is considered along with communication time delays between the agents.

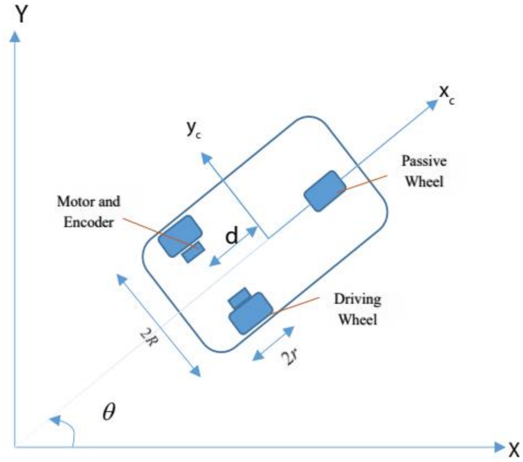



**Figure 1**

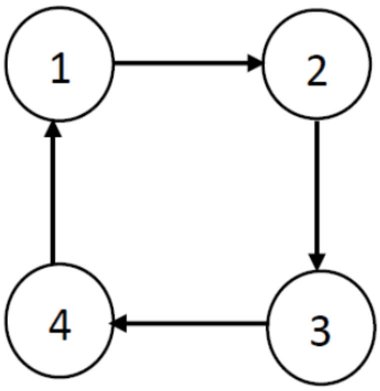
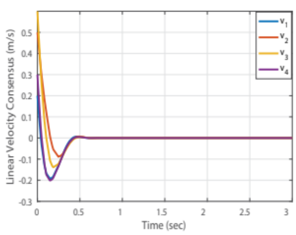
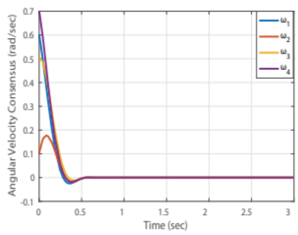


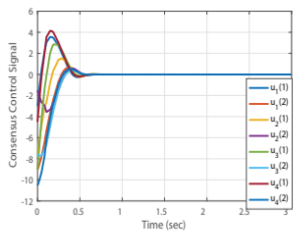

**Figure 2**



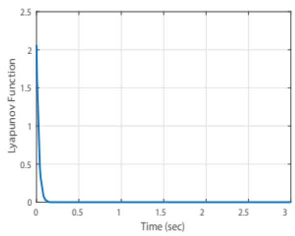

(a) Consensus oflinear velocities




(b) Consensus of angular velocities

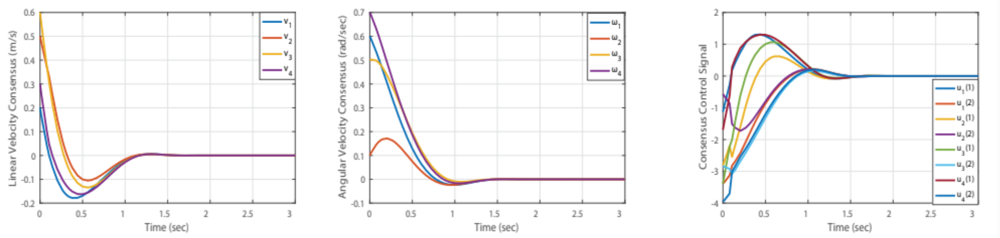



(c) Consensus control signals



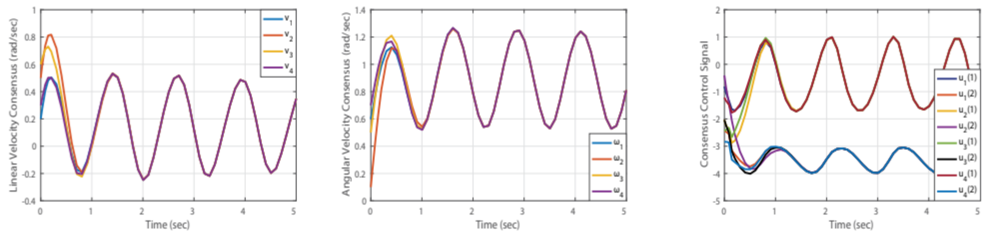

(d) Lyapunov function

**Figure 3**




(a) Consensus oflinear velocities (b) Consensus control signals (c) Consensus control signals

**Figure 4**




(a) Consensus oflinear velocities (b) Consensus control signals (c) Consensus control signals

**Figure 5**

The simulation results are shown in Figure 3, Figure 4 and Figure 5. In Figure 3, full access of the agents to their neighbor agent signals is considered and linear and angular velocities consensus, control signals and evolution of the Lyapunov function are shown in Figure 3a, Figure 3b, Figure 3c and Figure 3d, respectively. In Figure 4, it is assumed that each agent only has accessibility to the delayed linear velocity signals of its neighbors and angular velocities of the neighbors are not available for them. Consensus of linear and angular velocities of agents and control signals in this case are presented in Figure 4a, Figure 4b and Figure 4c, respectively. Besides, in Figure 5, the dynamic finite-time consensus condition in which the agents reach to the same velocities, with respect to a time-varying reference signal, and keep on moving with them are provided. Results show a good convergence rate to the desired values. It is worth mentioning that all scenarios are simulated with the delay-dependent finite-time consensus criterion. As mentioned before, the delay-independent one brings conservative results especially in the presence of small and known delays.

## 4. Conclusions

In this paper, the problem of finite-time consensus of a class of nonlinear systems in the presence of communication delays was considered. To compensate for the adverse effects of time-delay in communication between agents in a finite time interval, a novel consensus algorithm with two approaches, i.e., delay-independent/dependent was proposed. The algorithms may be used for cases with partial access of agents to their neighbor signals. Besides, there are no strong restrictions on the communication time-delays. Furthermore, the results of simulations on a group of mobile robot agents confirmed the theoretical findings. Future works include designing robust approaches with respect to the agents dynamics and considering Leader-follower scenarios and time-varying graph topologies.

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